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# Why Nature has made a choice of one time and three space coordinates?

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## Abstract

We propose a possible answer to one of the most exciting open questions in physics and cosmology, that is, the question why we seem to experience four-dimensional spacetime with three ordinary and one time dimensions. Making assumptions (such as particles being in first approximation massless) about the equations of motion, we argue for restrictions on the number of space and time dimensions. Accepting our explanation of the spacetime signature and the number of dimensions would be a point supporting (further) the importance of the ‘internal space’.

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## 1. Introduction

There are many experiences, prejudices so deeply embedded in our language and way of thinking, and so strongly connected with the idea that we have just one time dimension, that they could be used as arguments for such an idea, like the argument that the ordering ‘before’ and ‘after’ makes sense presupposing that there is just one time dimension. However, all these concepts and prejudices and experiences do not constitute a genuine explanation of why we were placed into just such a world. The main point of the present work is to discuss a more microphysical explanation for features of the numbers of space and time dimensions [1–17] by associating these numbers with properties of the equations—‘equations of motion’—obeyed by the fields of the elementary particles, especially involving what we can call the ‘internal space’, by which we mean the space of spins and charges so that we can think of all the different particles, which are fermions—such as quarks and leptons (and equivalently for bosons), as being different internal states of the same particle.

Theories of strings and membranes [18] and Kaluza–Klein-like theories [19] (as well as the approach by one of us [20–22], which unifies spins and charges in the space of anticommuting coordinates and predicts the connection between the spacetime dimension and the internal

degrees of freedom) predict initially more-than-four-dimensional spacetime. If this is true, how and when did our Universe in its evolution choose the Minkowski metric, and when and in which way did it ‘decide’ to (‘mostly’) manifest in four-dimensional spacetime out of a  $d$ -dimensional one, which could be any, even infinite?

- (i) In this paper we answer the question of how the *internal space* may result in restrictions on the choice of the signature of spacetime in any  $d$ -dimensional space, assuming *that the equations of motion operator exists (at least one) which is Hermitian, linear in the  $d$ -momentum  $p^a$  and has the eigenvectors in the internal space within the irreducible representation of the Lorentz group*. All these assumptions seem very mild: in standard quantum mechanics, Hermiticity of the Hamiltonian and thereby of the equations of motion operator guarantees a real value for the energy, unitarity in the time-development of a system and conservation of probability. It has been proved [23] that in even-dimensional spaces massless particles, respecting the Poincaré symmetry, obey equations of motion which are linear in the  $p^a$ -momentum. Siegel and Zwiebach 1987 [24] inform us that, starting from conformal symmetry, they came to the same conclusion for any dimension. The requirement that solutions of the equations of motion operator should be a linear superposition of the minimum number of basic states of the Lorentz group possibly leads to a choice of operators which the operator of equations of motion should commute with. We select in this paper the operator of handedness, which is the Casimir of the Lorentz group in any dimensional space. Massless particles, obeying the equations of motion, are in any dimensional space either left- or right-handed. (In [25], we also made a choice of the antilinear operators of charge conjugation, leading to Majoranas and quaternions.) Handedness seems to play a fundamental role in Nature: one of the assumptions of the standard model, dictated by experiment, is that only massless spinors of left-handedness carry a weak charge, while right-handed fermions are weak chargeless. Because of this property, spinors remain (almost) massless on the Planck scale. In other words, the Dirac equation for massive particles can only have solutions within the space of left- and right-handed states in  $(1 + 3)$ -dimensional space. According to the standard model, spinors receive small (observable) masses by interacting with the Higgs fields, which (by assumption) give the weak charge to the right-handed particles and accordingly also enable masses of the order of only the weak scale. This is known as the mass protection mechanism.
- (ii) In this paper we draw attention to a fascinating property that the mass protection mechanism only occurs in even-dimensional spaces. In odd-dimensional spaces, solutions of equations of motion span the same space for massless particles as they do for massive particles [26]. Therefore, the procedure of excluding part of the Hilbert space does not work, and accordingly no mass protection mechanism can occur. All spinors could accordingly acquire large masses—say the Planck mass. No interaction with, for instance, the Higgs is required to assure them a mass. But we also have no reason to believe that this mass is a very small one in comparison with the Planck scale. Spinors in odd-dimensional spaces could be invisible at ‘low’—experimentally accessible—energies.
- (iii) In this paper we argue that the stability of the equations of motion against a possible break of the Lorentz invariance privileges four-dimensional space (in addition to the two-dimensional one). (We are not saying that other dimensions are completely excluded.)

We consider only free (non-interacting) fields. Apart from momentum degrees of freedom, we also consider spin degrees of freedom and, for the sake of simplicity and transparency of presentation of our proofs and of the consequences of the proofs, we treat only spinors.

For a general case of any spin, the reader should see [4]. We would, however, like to point out that, using the Bargmann–Wigner (1934) prescription [27], any spin state can be constructed out of spinor states and accordingly all the requirements regarding the signature of the metric presented for spinors should be in agreement with the requirements of all other spin particles.

The ‘working hypotheses’ of the paper is that the fundamental space is  $d$ -dimensional, with  $d$  being any integer number, perhaps even infinite. In this paper we seek arguments which might have ‘led’ Nature to ‘end up’ with (‘mostly’, that is effectively,) four-dimensional spacetime, with one time and three space dimensions. We do not discuss any mechanism which would lead from some large (or even infinite) dimension down to four dimensions. Such a mechanism is certainly needed in theories such as string theories and Kaluza–Klein theories, as well as in the approach taken by one of us, which assumes that spins in  $d$ -dimensional space manifest in four-dimensional subspace as spin and all the known charges. We also do not claim that the proposed assumptions and their argued consequences are the only reasons for the ‘chosen’ dimension and signature of our observed world. But since the assumptions are rather mild ones, we expect the conclusions of this paper to be valid for any theory. Although we treat only massless non-interacting fields (of any spin), we expect that interactions among fields will not alter the conclusions, at least not for those types of interactions which can be switched on perturbatively. Further assumptions could, of course, further limit the allowed signatures.

## 2. Equations of motion

Massless spinors, if they preserve the Poincaré symmetry, obey *equations of motion, which are linear in the momentum  $d$ -vector* as proved in [23, 24]. We shall use somewhat generalized equations of motion of the type

$$(f^a(S^{cd})p_a)|\psi\rangle = 0 \quad (1)$$

with  $a = 0, 1, 2, 3, 5, \dots, d$ , where  $f^a$  is for each  $a$  any function of the generators of the Lorentz transformations  $S^{ab}$  in the *internal space* of spin degrees of freedom.

We, however, do not require that equation (1) is the only one that a spinor field must obey.

The total generators of the Lorentz transformations would read  $M^{ab} = L^{ab} + S^{ab}$ , with  $L^{ab} = x^a p^b - x^b p^a$ , which is the generator of the Lorentz transformations in ordinary space. In order for the equations of motion operator to be linear in  $p^a$ ,  $f^a$  can depend only on  $S^{ab}$ , that is on ‘internal space’. Both  $L^{ab}$  and  $S^{ab}$ , as well as their sum, fulfil the Lorentz algebra:  $[M^{ab}, M^{cd}]_- = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad})$ , where  $\eta^{ab} = \text{diag}\{\eta^{00}, \eta^{11}, \dots, \eta^{dd}\}$  is a (not yet specified) metric tensor with  $(\eta^{aa})^2 = 1$ , for each  $a$ . (In four-dimensional space the two well-known linear equations of motion in the momentum for free fields are the Dirac (or equivalently the Weyl) equation for spin  $\frac{1}{2}$  fermions (spinors) and the Maxwell equations for gauge Yang–Mills of spin 1 fields.)

Since this paper considers only spinors, the generators  $S^{ab}$ , which for spinors also fulfil the equation  $\{S^{ab}, S^{ac}\}_+ = \frac{1}{2}\eta^{aa}\eta^{bc}$ , can be expressed in terms of the operators  $\gamma^a$ ,  $a = 0, 1, 2, 3, 5, \dots, d$  (operating again in ‘internal space’) fulfilling the Clifford algebra

$$\{\gamma^a, \gamma^b\}_+ := \gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} \quad (2)$$

as follows

$$S^{ab} = \frac{i}{4}[\gamma^a, \gamma^b]_- := \frac{i}{2}\{\gamma^a\gamma^b - \eta^{ab}\}. \quad (3)$$

The generalized linear equations can then be written in the form

$$\mathcal{D}(\gamma^b)\gamma^a p_a = 0. \quad (4)$$

(The reader should recall that the Dirac equation for massless spinors is usually written in the form  $\gamma^a p_a = 0$ .) The Hermiticity condition reads

$$\gamma^{a\dagger}(\mathcal{D}(\gamma^b))^\dagger = \mathcal{D}(\gamma^b)\gamma^a \quad (5)$$

since the operator  $p^a$  is a Hermitian one, if the usual inner product in ordinary space is assumed. The requirement of Hermiticity without allowing for an extra *internal space* matrix  $\mathcal{D}(\gamma^b)$  would be too strong a requirement. Performing Hermitian conjugation of equation (2) and requiring that the inner product of a ket  $\gamma^a|\psi\rangle$  and a bra  $(\gamma^a|\psi\rangle)^\dagger$  has to have the same value as  $\langle\psi|\psi\rangle$  which leads to the unitarity condition  $\gamma^{a\dagger}\gamma^a = I$ , for any  $a$ , we find

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a \quad (6)$$

and accordingly  $S^{ab\dagger} = \eta^{aa}\eta^{bb}S^{ab}$ . According to equations (6) and (5) the Hermiticity condition for the equations of motion operator reads:

$$(\mathcal{D}(\gamma^b))^\dagger = \gamma^a\mathcal{D}(\gamma^b)\gamma^a \quad \text{for each } a. \quad (7)$$

We define, according to [21, 23], the operator  $\Gamma$ , which in even-dimensional spaces determines the handedness of states for any spin. In this paper we shall express the operator of handedness in terms of  $\gamma^a$ , since we treat only spinors ( $\Gamma$  is for  $d = 4$  and for spinors known as  $\gamma^5$ ). It then has meaning for any dimensional space:

$$\Gamma = \prod_a (\sqrt{\eta^{aa}}\gamma^a) \begin{cases} (i)^{\frac{d}{2}} & \text{for } d \text{ even} \\ (i)^{\frac{d-1}{2}} & \text{for } d \text{ odd} \end{cases} \quad (8)$$

and the product of  $\gamma^a$  is assumed to be in the rising order with respect to index  $a$ . We chose the phase such that the operator  $\Gamma$  is Hermitian and its square is the unit operator

$$\Gamma^\dagger = \Gamma \quad \Gamma^2 = I. \quad (9)$$

We then easily find that

$$\{\Gamma, \gamma^a\}_\pm = \begin{cases} 0 & \text{for } + \text{ sign and } d \text{ even} \\ 0 & \text{for } - \text{ sign and } d \text{ odd} \end{cases} \quad (10)$$

and that  $\Gamma$  is a Casimir of the Lorentz group, i.e.  $\{\Gamma, S^{ab}\}_- = 0$ .

### 3. Reducibility of representations

Equations (4) and (7) concern the linearity and Hermiticity requirements for the operator of equations of motion. Only the *reducibility* has, according to our assumptions, yet to be taken into account. Since we assume the operator of handedness to take care of reducibility, we accordingly require that the operator of equations of motion and the operator of handedness commute

$$\{\Gamma, \mathcal{D}\gamma^a p_a\}_- = \{\Gamma, \mathcal{D}\gamma^a\}_- = 0. \quad (11)$$

The last equation has to be fulfilled for each  $a$ . We multiply this equation from the left by  $\Gamma$  and take into account the properties of  $\Gamma$  ( $\Gamma^\dagger = \Gamma$  and  $\Gamma^2 = I$ , equation (9)). It follows then, if we also take into account equation (10) (which states that in even-dimensional spaces  $\Gamma$  anticommutes with  $\gamma^a$ , while in odd-dimensional spaces they commute) that

$$\begin{aligned} \mathcal{D}\gamma^a + \Gamma^{-1}\mathcal{D}\Gamma\gamma^a &= 0 & \text{for } d \text{ even,} \\ \mathcal{D}\gamma^a - \mathcal{D}\gamma^a &= 0 & \text{for } d \text{ odd.} \end{aligned} \quad (12)$$

We conclude that in odd-dimensional spaces the reducibility requirement leads to no limitation whatsoever on the signature of the metric. We find that  $\Gamma^{-1}\mathcal{D}\Gamma = (\gamma^d)^{-1}(\gamma^{d-1})^{-1}\dots(\gamma^0)^{-1}\mathcal{D}\gamma^0\dots\gamma^{d-1}\gamma^d$ . Using (7) and its Hermitian conjugate ( $\mathcal{D} = \gamma^a\mathcal{D}^\dagger\gamma^a$ ) we find by repetition that  $\Gamma^{-1}\mathcal{D}\Gamma = \mathcal{D}\prod_a\eta^{aa}$ . In order to fulfil equation (12) for even  $d$  it follows that  $\prod_a\eta^{aa} = -1$ .

In even-dimensional spaces the requirement is severe: solutions of equations of motion can only have well-defined handedness in spaces of odd-time and odd-space signatures. In a four-dimensional space it means that only one time and three space signature is possible (or the reverse, although this is not important, as the overall sign is not important).

In four dimensions our result,  $q$  being odd, means that we must have either three time dimensions and one space dimension or, as we know, we have the reverse.

#### 4. Massless and massive solutions and the mass protection mechanism

We shall prove that the so-called mass protection mechanism occurs only in even-dimensional spaces. To prove this, we look at the properties wrt irreducibility of an equation of motion operator with a mass term as follows:

$$(\gamma^a p_a - m)|\psi\rangle = 0. \quad (13)$$

This is for  $d$  equal to four and for the Minkowski metric the well-known Dirac equation, with an operator which is not Hermitian. If we want the operator to be Hermitian, we multiply it by an appropriate matrix  $D$  as above (which in the Dirac equation case is  $\gamma^0$ ). We assume the equation to be valid for any dimension  $d$ . Of course, for the mass protection discussion we cannot retain the assumption of linearity, since the mass term is obviously not linear in momentum. However, we do insist on the irreducibility assumption. Let us check whether the irreducibility requirement is at all possible with the mass term added. We use the fact that irreducible representations are obtained for the Dirac equation by projection with matrices  $\frac{1}{2}(1 \pm \Gamma)$  where  $\Gamma$  is given by (8), i.e. we restrict the state space of the Dirac equation for any  $d$  and any signature to those states that obey

$$D(\gamma^a p_a - m)\frac{1}{2}(1 - \Gamma)|\psi\rangle = 0 \quad (14)$$

$\Gamma|\psi\rangle = |\psi\rangle$  say, so that  $|\psi\rangle = \frac{1}{2}(1 - \Gamma)|\psi\rangle$ .

For the projected Dirac equation to be irreducible, we must require that the equation (14) maps into the same subspace to which  $\frac{1}{2}(1 - \Gamma)|\psi\rangle$  belongs. The requirement of these operators mapping onto the space projected by  $\frac{1}{2}(1 - \Gamma)$  can be expressed by the requirement that projection by the projection operator  $\frac{1}{2}(1 + \Gamma)$  on the orthogonal space will give zero regardless of the state  $|\psi\rangle$ . We accordingly require

$$\frac{1}{2}(1 + \Gamma)D(\gamma^a p_a - m)\frac{1}{2}(1 - \Gamma) = 0 \quad (15)$$

for all values of  $p_a$ , on the operator level (which means that equation (15) is not the equation of motion here) which leads to

$$[\Gamma, Dm] = 0 \quad \text{and} \quad [\Gamma, D\gamma^a] = 0. \quad (16)$$

Combining these two equations with equation (10) we see that it is impossible to be satisfied in even dimensions  $d$ . In even dimensions the requirement of irreducibility prevents the mass term occurring since the only way out is to take the mass  $m = 0$ . In the case of odd dimensions, any mass is allowed even after  $\Gamma$ -projection. This prevention of mass—in even  $d$ —is what is called mass protection, in the sense that a theory, if arranged in such a way that its symmetries (charges etc) enforce only the  $\Gamma$ -projected state space to be used, then

that theory can explain why the particles in question are massless, since a theory does not allow solutions of the equations of motion for massive spinors to exist, because the part of the space which has opposite handedness is missing. In the standard model (see [28]) the Weinberg–Salam–Higgs field at the end gives (by ‘hand’) most fermions a ‘little’ mass by breaking the gauge symmetries which caused the mass protection.

In odd dimensions  $d$ , however, it is not possible to prevent spinors from acquiring a mass, because non-zero mass is allowed even when the  $\Gamma$ -projection is performed: the massive and massless Dirac equations have solutions within the same space of states. (Only the coefficients change but none of the states disappear.) Taking the point of view that all parameters, say the mass, not forbidden are present with a scale of size given by an order of magnitude of a fundamental scale assumed to be very big—say the Planck scale—compared to energies per particle accessible in practice, we conclude that in odd dimensions all spin one-half particles will for practical purposes acquire such large masses that they effectively cannot be observed. *So odd dimensions deviate from even ones by typically having all the masses of the fundamental scale, while in even dimensions the mass protection mechanism is possible.*

## 5. Stability of the equations of motion leading to dimensions being at most four

We shall add to our assumptions about the equations of motion operator in sections 2 and 3 the requirement that the equations of motion should be stable in the sense that if we *infinitesimally destroy the Lorentz invariance* by adding a small extra term still obeying the other assumptions, this term would in reality not disturb the equations of motion in the following sense: we can transform it away by shifting the coordination of the momentum  $p_a$  and changing the metric tensor  $\eta^{ab}$  into a new set of values. This argumentation is really a rewriting of the old argument of ‘random dynamics’ of one of us [8].

Let us in fact imagine that the equations of motion (4) with abstract notation  $f^a = \mathcal{D}\gamma^a$  are modified slightly from  $f^a p_a |\psi\rangle = 0$  into

$$(f^a + f'^a) p_a |\psi\rangle = 0. \quad (17)$$

Can we then pretend that this new equation is indeed just of the same form as before, but with slightly changed notation for the way one expands the momentum on the basis vectors and the metric tensor  $g^{ab}$  known from general relativity? In other words, can we write the modified equations in a slightly more general form by changing the basis for the  $d$ -momentum  $p_a$

$$g^{ab} f_a p_b |\psi\rangle = 0? \quad (18)$$

By counting degrees of freedom, we see that this could only be the case for a general modification term  $f'^a p_a$  provided we have at most four dimensions, i.e. it is at least needed that  $d \leq 4$ .

This counting goes as follows: the number of degrees of freedom of the extra term  $f'^a p_a$  is that of  $d$  matrices with the number of columns and rows equal to the dimension of the irreducible representation of the ‘Weyl’, which is  $2^{d/2-1}$  for an even dimension  $d$  and  $2^{(d-1)/2}$  for an odd dimension  $d$ . Altogether, this means that the number of real parameters in the modification is  $d \cdot 2^{d-2}$  for even and  $d \cdot 2^{d-1}$  for odd  $d$ . These modification parameters should be compensated by  $d(d+1)/2$  parameters in the metric tensor  $g^{ab}$  (or  $\eta^{ab}$ ) and  $d(d-1)/2$  parameters associated with making a Lorentz transformation of the basis for the  $d$ -momentum  $p_a$ . We need accordingly the inequality

$$d(d+1)/2 + d(d-1)/2 \geq d \cdot 2^{d-2} \quad (19)$$

for even  $d$ , and

$$d(d+1)/2 + d(d-1)/2 \geq d \cdot 2^{d-1} \quad (20)$$

for odd  $d$ . These equations reduce to  $d \geq 2^{d-2}$  and  $d \geq 2^{d-1}$  for even and odd  $d$  respectively. Thus we must for even  $d$  have  $d \leq 4$ , while for odd  $d$  only  $d = 1$  satisfies the inequality.

Accordingly, this stability requirement can only work for  $d = 1, 2$  and 4 dimensions.

### 5.1. Almost inconsistency concerning Lorentz invariance

We have to admit that with the requirement that the equations of motion should be stable against an infinitesimal destruction of the Lorentz invariance just discussed, we are *a priori* really assuming an inconsistency with respect to the fact that in the rest of the paper the Lorentz invariance has been assumed. (We are grateful to the referee for pointing out this problem.) There is, however, strictly speaking the little loophole that Lorentz invariance could be a *non-valid principle at the fundamental level*, but that it would nevertheless come out phenomenologically at some low energy level or under special conditions. This is in fact the idea which one of us has speculated in [5] and in some of the related references [5–9]. Further developments of this type of hope of Lorentz invariance and also other symmetries coming out by themselves can be found in the collection of papers [13], where we seek to justify that the Lorentz invariance can be hoped to come out sufficiently accurately when built into the gravity theory.

## 6. Conclusion

In this paper we sought arguments which might have ‘guided’ Nature to ‘mostly’ (effectively) manifest in four-dimensional ordinary space, with one time and three space coordinates, in addition to the ‘internal space’ of spins and charges. We argued that it is the ‘internal space’ which forced Nature to ‘make a choice’ of  $d$  equal four, if rather mild assumptions about the properties of the operator of equations of motion for spinors are the meaningful ones we believe they are. (Further assumptions would further limit possible signatures in  $d$ -dimensional space. While a somewhat relaxed stability assumption could tell, for example, why spin degrees of freedom in higher than four dimensions might demonstrate as (conserved) charges in four-dimensional spacetime.) These mild assumptions lead to various predictions concerning the spacetime dimensions. The five assumptions which we used were:

- (1) linearity in momentum,
- (2) Hermiticity,
- (3) irreducibility,
- (4) mass protection,
- (5) stability.

We proved that, if we apply assumptions (1), (2) and (3) in *even-dimensional spaces*, only *odd-time and odd-space dimensions are possible*, while in *odd-dimensional spaces all signatures are possible*. However, while in *even-dimensional spaces limitation to only one irreducible representation, for example left handedness, enables fermions to remain massless*, this is not true for *odd-dimensional spaces*, since the solutions for a massless and a massive case span the same space. So applying assumption (4) we *exclude an odd number of space plus time dimensions*.

Applying assumptions (1), (2), (3) and (5), we concluded that the total dimension should be 1, 2 or 4. Taking account of assumption (4), we exclude  $d = 1$  and using the odd-time and odd-space dimensions we finally obtain from all our five assumptions only the time–space



dimensions  $1 + 1$  or  $1 + 3$  (or the opposite). This is thus close to explaining the experimental numbers  $1 + 3$ .

We understand the time-space dimension  $1 + 3$  as an effective dimension.

Further studies along these lines might involve consideration of different kinds of representations, such as Majoranas (the paper by the two authors of this paper, entitled ‘The internal space is making the choice of the signature of spacetime’, which also considers Majoranas, is almost ready for publication), which are representations with real coefficients (i.e. using the field of real numbers). However, it turns out that these kinds of representations have no mass protection mechanism either, and the corresponding fields are accordingly invisible at low energies.

This paper treats only free (non-interacting) fields of any spin in  $d$ -dimensional space of any signature and any  $d$ . However, we expect that interactions among fields will not alter the conclusions of this paper, that is they do not result in additional limitations on possible signatures, if only those types of interactions which can be switched on perturbatively are assumed.

Allowing any dimension  $d$  with ordinary and spin degrees of freedom, we only look for properties of equations of motion (we treated one equations of motion operator, but we do not require at all that it must be only one), which could possibly be responsible for four ordinary effective (detectable) dimensions—in addition to ‘internal space’. We did not consider mechanisms, which could have brought  $d$  ordinary (and internal) dimensions to four effective dimensions, or the possibility (proposed by the approach of one of the authors [20–22] of this paper) that spins of higher than four dimensions are responsible for charges in the effective  $(1 + 3)$ -dimension. We also do not comment on where the limitation that only states of one handedness (which leads to the mass protection mechanism for even-dimensional spaces) could come from or where the assumption that the Higgs mechanism brings the weak charge to right-handed spinors and accordingly enables masses to spinors on some (weak) scale, comes from.

(By the number of times we have here meant the number of dimensions with a certain signature of the metric. But there is another way in which one could define something that with some right could be called the number of time dimensions—namely, the number of equations. One could consider more than one equation of motion per field component. In the present paper we indeed discuss only one equation of motion operator per field component. And this could be understood in this sense that we did already assume indirectly only one time. But we have never required at all that the discussed equations of motion operator is the only one that fields must be the eigenfields of. If we would require only one equation of motion operator, it could have in this different sense be assumed just one time dimension. This comment concerns indeed the unitarity (or antiunitarity) properties of the non-compact Lorentz group for any non-Euclidean signature and accordingly the definition of the inner product, which for the  $1 + 3$ , and equivalently also for the  $1 + d - 1$ , case resembles  $\langle \psi | \gamma^0 \psi \rangle$ . In this paper, we leave the problem of a possible definition of the inner product for the case of more than one time coordinate open—we could namely for  $q + (d - q)$ , with  $q$  time-like coordinates, define the inner product as  $\langle \psi | \mathcal{O} \psi \rangle$ , with  $\mathcal{O}$  proportional to the product of all the  $\gamma^a$  with time-like signature. Only if one accepts that not more than one equation of motion operator for each field component exists and the inner product with  $\gamma^0$  is accepted, could one say that we put in a different meaning that here we have just one and thus especially an odd number of times.)

Yet the results of our conclusions, if the (rather mild) assumptions can be taken seriously (which we believe they should), are conclusive. They are also restrictive for theories with additional degrees of freedom, such as string theories and Kaluza–Klein theories.

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## References

- [1] Hartle J and Hawking S W 1983 *Phys. Rev. D* **28** 2960
- [2] Bouso R and Hawking S W 1998 Lorentzian condition in quantum gravity *Preprint* hep-th/9807148
- [3] Tegmark M 1997 On dimensionality of spacetime *Class. Quantum Grav.* **14** L69–75 (*Preprint* gr-qc/9702052)
- [4] Mankoč Borštnik N S and Nielsen H B 2000 Why odd-space and odd-time dimensions in even-dimensional spaces *Phys. Lett. B* **468** 314–21
- [5] Nielsen H B 1976 Dual strings—section 6. Catastrophe theory programme *Fundamentals of Quark Models (Scottish Universities Summer School in Physics)* ed I M Barbour and A T Davies pp 528–43
- [6] Nielsen H B 1983 *Gauge Theories of the Eighties* ed R Raitio and J Lindfors (New York: Springer) p 288
- [7] Nielsen H B, Bennett D L and Brene N 1985 *Recent Developments in Quantum Field Theory* ed J Ambjørn, B Duurhus and J L Petersen (Amsterdam: Elsevier) p 253
- [8] Nielsen H B and Rugh S E 1992 Weyl particles, weak interactions and origin of geometry *Nucl. Phys. B* **29** 200–46
- [9] Nielsen H B and Rugh S E 1992 Why do we have 3 + 1 dimensions *Wendisch-Rietz Three-Dimensional Space-Time Meeting* ed B Dörfel and W Wiecezorek (*Preprint* hep-th/9407011)
- [10] Wheeler J 1983 Law without law *Quantum Theory of Measurement* ed J A Wheeler and W R Zurek (Princeton, NJ: Princeton University Press) p 182
- [11] Chew G F 1973 *Properties of Fundamental Interactions* ed E Zichichi (Editrice Compositori) p 3
- [12] Woo C H 1999 Mission impossible? A look at past setbacks in the search for elementary matter and for universal symmetries *Preprint*
- [13] Froggatt C D and Nielsen H B 1996 *Origin of Symmetries* (Singapore: World Scientific)
- [14] Greensite J 1993 Dynamical origin of the Lorentzian signature of spacetime *Phys. Lett. B* **300** 34–7
- [15] Carlini A and Greensite J 1994 Why is space-time Lorentzian? *Phys. Rev. D* **49** 866–78
- [16] Weinberg S 1986 *Proc 23rd Int. Conf. on High Energy Physics (Berkeley)* (Singapore: World Scientific) p 217
- [17] Penrose R and Rindler W 1986 *Spinors and Space-Time* (Cambridge: Cambridge University Press) p 235
- [18] Kaku M 1998 *Introduction to Superstrings* (New York: Springer)
- [19] Li H C (ed) 1984 *An Introduction to Kaluza–Klein Theories* (Singapore: World Scientific)
- [20] Mankoč Borštnik N S 1992 Spin connection as a superpartner of a vielbein *Phys. Lett. B* **292** 25–9
- [21] Mankoč Borštnik N S 1993 Spinor and vector representations in four dimensional Grassmann space *J. Math. Phys.* **34** 3731–45
- [22] Mankoč Borštnik N S 2001 Unification of spins and charges *Int. J. Theor. Phys.* **40** 315–37 and references therein
- [23] Gornik B and Mankoč Borštnik N S 2001 Linear equations of motion for massless particles of any spin in any even-dimensional spaces *Preprints* hep-th/0102067, hep-th/ 0102008
- [24] Siegel W and Zwiebach B 1987 Gauge string fields from light cone *Nucl. Phys. B* **282** 125–41
- [25] Mankoč Borštnik N S and Nielsen H B 2002 The internal space is making the choice of the signature of space-time in preparation
- [26] Mankoč Borštnik N S and Nielsen H B 2002 How to generate spinor representations in any dimension in terms of projection operators *J. Math. Phys.* **43** 5782
- [27] Bargmann V and Wigner E P 1948 *Proc. Natl Acad. Sci. USA* **34** 211
- [28] Cheng T P and Li L F 1986 *Gauge Theories of Elementary Particle Physics* (Oxford: Oxford University Press)